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Anti-controlling quasi-periodic impact motion of an inertial impact shaker system

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ABSTRACT

In this paper, anti-controlling quasi-periodic impact motion of an inertial impact shaker system is addressed. There exist two aspects of difficulty in the anti-control design: one is from the implicit Poincaré map of the system itself and the other from the limitation of the classical critical criterion of Hopf bifurcation described by the properties of eigenvalues. Through using linear feedback control method in the original differential system and applying an explicit criterion of Hopf bifurcation without using eigenvalues to the Poincaré map of the close-loop system, the two difficulties above can be overcome and the control design for creation of the quasi-periodic impact motion at a specified system parameter location is achieved. Numerical simulation shows that the stable quasi-periodic impact motion of the system is created at a desired parameter location by adjusting control parameter appropriately.

1. Introduction

There are a lot of impact mechanical devices whose operation principle is based on vibro-impact dynamics, such as inertial impact shakers, impact vibration dampers and gears. Researches into the dynamic mechanism of vibro-impact systems are of great significance for stability and reliability analysis, noise suppression and optimum designs. The periodic motions of two-degree-of-freedom vibratory system with multiple constraints were investigated by Wagg and Bishop [1]. The Hopf bifurcations of Poincaré map for vibro-impact systems and the existence of quasi-periodic impact motion were studied in Refs. [2,3]. In the literature [4–7], several kinds of codimension two Hopf bifurcation phenomena of vibro-impact systems are reported, including the degenerate Hopf bifurcation, interaction of Hopf and period doubling bifurcations, interaction of Hopf–Hopf bifurcation.

Bifurcation control has attracted many researchers' attentions [8–11]. In general, the goal of bifurcation control is to modify the bifurcation characteristics of a nonlinear system including delaying the onset of an inherent bifurcation [12] and modifying the amplitudes [13] and stability [14] of bifurcated solutions. By contrast, anti-control of bifurcation, as the "inverse" problem of conventional bifurcation analysis, is aimed at creating a certain bifurcation with desired dynamic properties at a specified system parameter location via control method. The main purpose of this paper is to address the problem of anti-control of Hopf bifurcation of the Poincaré map of an inertial impact shaker system (i.e., the second Hopf bifurcation of the original system), which may be viewed as a design approach to create a quasi-periodic impact motion (or torus solution) at a specified system parameter location via control. It should be mentioned that as the bifurcation

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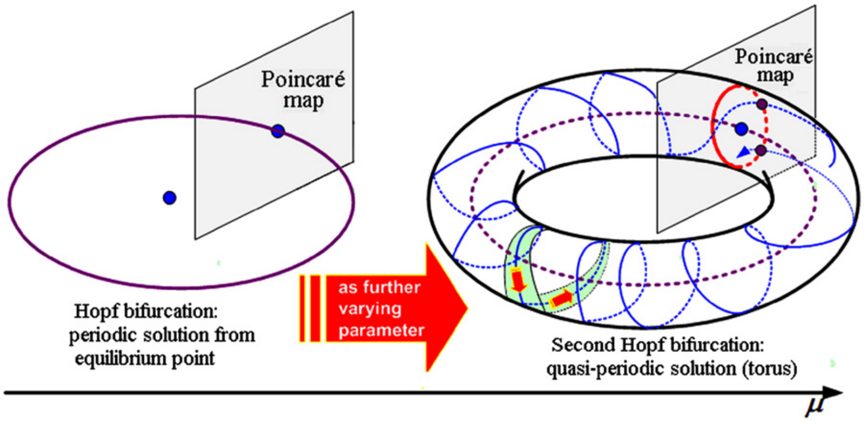

Fig. 1. Hopf bifurcation results in periodic solution whereas second Hopf bifurcation gives rise to quasi-periodic solution (torus).

solution of the second Hopf bifurcation, the bifurcated Hopf limit circle of the Poincaré map of a vibro-impact system corresponds to the quasi-periodic impact motion of the original vibro-impact system (as shown in Fig. 1).

In order to create the quasi-periodic impact motion, it is required that the controller must be set up in the original differential system but its gains need to be determined by the bifurcation criterion of the corresponding Poincaré map. In general, there exist two aspects of difficulty in anti-control design of quasi-periodic impact motion. One is the difficulty from the implicit Poincaré map of inertial impact shaker system. Notice that the analysis of quasi-periodic solution is usually equivalent to the analysis of Hopf bifurcation of Poincaré map (the second Hopf bifurcation of the original differential system). However, the Poincaré map of an inertial impact shaker system is of implicit form subject to the noncontinuity property of repeated impacts. If we use the control method based on the Poincaré map of the vibro-impact system, it is very difficult to deduce the control gains of the original differential system and to achieve anti-control of the system in real implement of control. Thus, without changing the original system's periodic solution, we will apply linear feedback control method in the original differential system instead of the Poincaré map. The other difficulty originates from the classical bifurcation critical criteria (or bifurcation definitions) described by the properties of eigenvalues. Different from the traditional bifurcation analysis, there exist multiple control parameters (gains) in the Jacobian matrix of the close-loop control system to be determined. In the quantitative analysis of gains, it is expected to obtain the analytical expressions of all eigenvalues with respect to the control parameters. However, it should be stressed that the analytical expressions of all eigenvalues for a non-constant matrix of high order are unavailable in general. This implies that with application of the classical bifurcation critical criteria, we have to numerically compute eigenvalues point by point and check their properties to search for the control gains. In order to overcome the difficulty, an explicit criterion of Hopf bifurcation for the map without directly using eigenvalues is used to obtain the gains and achieve the goal of creating the quasi-periodic impact motion at a specified system parameter location. In addition, the stability of the created quasi-periodic solution is analyzed in detail. Numerical simulation shows that the stable quasi-periodic impact motion of the inertial impact shaker system is created in the pre-specified parameter location.

2. Inertial impact shaker system and its periodic motion

2.1. Mechanical model of inertial impact shaker

The inertial impact shaker model is shown schematically in Fig. 2. A vibrating platform with mass M is connected to the foundation with a linear spring with stiffness K and a linear viscous dashpot with damping constant C. The platform is subjected to a harmonic excitation with amplitude F_0, excitation frequency ω and phase angle δ. The rigid-body cast with mass m is in the gravitation field without other forces when no impact occurs. Consequently, the cast bounces on the flat horizontal surface of the platform. Let Y and X denote the displacements of mass m and mass M, respectively. The mass m will impact with the mass M while the bottom surface of the former contacts with the top surface of the later (i.e., $Y=X$) at a non-zero relative velocity.

The impact dynamics of the inertial impact shaker system can be described by the following Eqs. (1) and (2).

$$\begin{cases} M\ddot{X}+C\dot{X}+KX=F_0\sin(\omega t+\delta) \\ \ddot{Y}=-g \end{cases}, \quad (X\neq Y) \tag{1}$$

$$\begin{cases} M\dot{X}_-+m\dot{Y}_-=M\dot{X}_++m\dot{Y}_+ \\ \dot{X}_+-\dot{Y}_+=-\hat{R}(\dot{X}_--\dot{Y}_-) \end{cases}, \quad (X=Y) \tag{2}$$

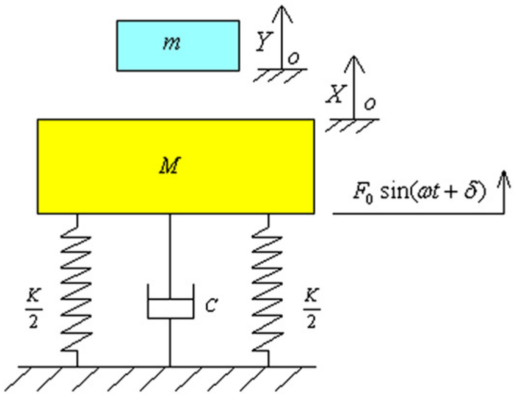

Fig. 2. Schematic of an inertial impact shaker model.

where $\dot{X}_-$ and $\dot{Y}_-$ are respectively the instant velocities of M and m at contact points just before impacts. $\dot{X}_+$ and $\dot{Y}_+$ are respectively the instant velocities of M and m at contact points just after impacts. $\hat{R}$ stands for the constant coefficient of restitution.

We transform the system (1) and (2) into the following non-dimensional form for convenience:

$$\begin{cases} \ddot{x}+\dfrac{2\zeta}{z}\dot{x}+\dfrac{1}{z^2}x=\dfrac{\sqrt{(1-z^2)^2+(2\zeta z)^2}}{z^2}\sin(\theta+\delta), & (x\neq y) \\ \ddot{y}=-e_1 \end{cases} \tag{3}$$

$$\begin{cases} \dot{x}_-+\mu\dot{y}_-=\dot{x}_++\mu\dot{y}_+ \\ \dot{x}_+-\dot{y}_+=-\hat{R}(\dot{x}_--\dot{y}_-) \end{cases}, \quad (x=y) \tag{4}$$

where

$$x=X/s,\quad y=Y/s,\ \zeta=C/2M\omega_n,\ z=\omega/\omega_n,$$
$$e_1=\sqrt{(1-z^2)^2+(2\zeta z)^2}/(\beta z^2),\ \beta=F_0/Mg,\ \mu=m/M,\ \theta=\omega t,$$
$$s=F_0/(K\sqrt{(1-z^2)^2+(2\zeta z)^2})\quad \text{and}\quad \omega_n=\sqrt{K/M} \tag{5}$$

We call the system before control described by Eqs. (1) and (2) or (3) and (4) as "the original system" and the system under control as "the controlled system" in later sections for convenience.

2.2. Periodic motion of inertial impact shaker system

As mentioned above, a Hopf limit circle bifurcating from a fixed point of the Poincaré map corresponds to the quasi-periodic impact motion in the original vibro-impact system. In order to create the quasi-periodic impact motion in the original system via control, Hopf bifurcation of a fixed point in the Poincaré map becomes our control objective. Notice that a fixed point of the Poincaré map stands for a periodic impact motion of the original system. Thus, we first discuss the existence of a periodic impact motion and its analytical expression, which will be used in the design procedures of anti-control of quasi-periodic impact motions in the next section.

To present a periodic impact motion of the original system, we need to give the full solutions of Eqs. (3) as follows:

$$x(\theta)=e^{-\zeta(\theta/z)}(b_1\cos\eta(\theta/z)+b_2\sin\eta(\theta/z))+\sin(\theta+\tau) \tag{6}$$

$$\dot{x}(\theta)=[e^{-\zeta(\theta/z)}/z][(b_2\eta-b_1\zeta)\cos\eta(\theta/z)-(b_2\zeta+b_1\eta)\sin\eta(\theta/z)]+\cos(\theta+\tau) \tag{7}$$

$$y(\theta)=b_3+b_4\theta-e_1\theta^2/2 \tag{8}$$

$$\dot{y}(\theta)=b_4-e_1\theta \tag{9}$$

where $\eta=\sqrt{1-\zeta^2}$, $\tau=\delta-\varphi$, $\varphi=\tan^{-1}[2\zeta z/(1-z^2)]$. The constants of integration b_1, b_2, b_3 and b_4 are determined by the initial condition and the parameters of the system.

The periodic impact motion of the original system (3) and (4) can be determined by the following set of periodicity and initial conditions:

$$x(0)=x(2\pi),\quad y(0)=y(2\pi),\quad x(0)=y(0) \tag{10a}$$

$$\dot{x}_+(0)=\frac{1-\mu\hat{R}}{1+\mu}\dot{x}_-(2\pi)+\frac{\mu(1+\hat{R})}{1+\mu}\dot{y}_-(2\pi) \tag{10b}$$

$$\dot{y}_+(0)=\frac{1+\hat{R}}{1+\mu}\dot{x}_-(2\pi)+\frac{\mu-\hat{R}}{1+\mu}\dot{y}_-(2\pi) \tag{10c}$$

Subject to the conditions in (10a)–(10c), we can obtain the constants of integration and phase in (6)–(9) as follows:

$$\tau_0=\cos^{-1}\left(e_1\pi\left(\frac{1-(2\mu+1)\hat{R}}{1+\hat{R}}-\frac{2\mu(\eta(c-1)+\zeta s)}{\eta(s^2+(c-1)^2)}\right)\right) \tag{11}$$

$$b_{10}=\frac{-2\pi z\mu s e_1}{\eta(s^2+(c-1)^2)},\quad b_{20}=\frac{2\pi z\mu(c-1)e_1}{\eta(s^2+(c-1)^2)} \tag{12}$$

$$b_{30}=b_{10}+\sin(\tau_0),\quad b_{40}=e_1\pi \tag{13}$$

where $s=e^{-\zeta(2\pi/z)}\sin\eta(2\pi/z)$, $c=e^{-\zeta(2\pi/z)}\cos\eta(2\pi/z)$. Therefore, the non-impact part of the periodic impact motion of the original system can be written as the following form:

$$x_p(\theta)=e^{-\zeta(\theta/z)}(b_{10}\cos\eta(\theta/z)+b_{20}\sin\eta(\theta/z))+\sin(\theta+\tau_0) \tag{14}$$

$$\dot{x}_p(\theta)=[e^{-\zeta(\theta/z)}/z][(b_{20}\eta-b_{10}\zeta)\cos\eta(\theta/z)-(b_{20}\zeta+b_{10}\eta)\sin\eta(\theta/z)]+\cos(\theta+\tau_0) \tag{15}$$

$$y_p(\theta)=b_{30}+b_{40}\theta-e_1\theta^2/2 \tag{16}$$

$$\dot{y}_p(\theta)=b_{40}-e_1\theta \tag{17}$$

After the periodic impact motion of the original system is obtained, we may establish the Poincaré map along with the periodic impact motion as follows [7]:

$$\tilde{\mathbf{X}}_{\mathbf{k+1}}=\mathbf{f}(\alpha,\tilde{\mathbf{X}}_{\mathbf{k}}) \tag{18}$$

where $\tilde{\mathbf{X}}_{\mathbf{k}}=(\tilde{x}_k,\dot{\tilde{x}}_k,\dot{\tilde{y}}_k,\tilde{\tau}_k)^{\mathrm{T}}$ and the real parameter $\alpha\in\mathrm{R}$. The periodic impact motion of the original system becomes a fixed point of the Poincaré map (18).

3. Anti-controlling of quasi-periodic impact motion

In this section, we design the linear feedback controller for anti-controlling quasi-periodic impact motion of the inertial impact shaker system. In what follows, an explicit critical criterion of Hopf bifurcation for map can be utilized to obtain the gains.

3.1. Inertial impact shaker system under linear feedback controller and its Poincaré map

The inertial impact shaker system under linear feedback controller is

$$\begin{cases} M\ddot{X}+C\dot{X}+U(\dot{X}-\dot{X}_p)+KX+V(X-X_p)=F_0\sin(\omega t+\delta)\\ \ddot{Y}=-g\end{cases} \tag{19}$$

and

$$\begin{cases} M\dot{X}_-+m\dot{Y}_-=M\dot{X}_++m\dot{Y}_+\\ \dot{X}_+-\dot{Y}_+=-\hat{R}(\dot{X}_--\dot{Y}_-)\end{cases} \tag{20}$$

where U and V are the linear control gains. $\dot{X}_p$ and X_p expressing the periodic solution are the dimensional form of the non-dimensional quantities $\dot{x}_p$ and x_p in (14) and (15).

The system (19) and (20) takes the following non-dimensional form:

$$\begin{cases} \ddot{x}+\dfrac{2\zeta}{z}\dot{x}+\dfrac{2u}{z}(\dot{x}-\dot{x}_p)+\dfrac{1}{z^2}x+\dfrac{v}{z^2}(x-x_p)=\dfrac{\sqrt{(1-z^2)^2+(2\zeta z)^2}}{z^2}\sin(\theta+\delta)\\ \ddot{y}=-e_1\end{cases} \tag{21}$$

$$\begin{cases} \dot{x}_-+\mu\dot{y}_-=\dot{x}_++\mu\dot{y}_+\\ \dot{x}_+-\dot{y}_+=-\hat{R}(\dot{x}_--\dot{y}_-)\end{cases} \tag{22}$$

where the non-dimensional quantities $u=U/2M\omega_n$ and $v=V/K$, the rest are the same in (5).

By suitable transformations, we can obtain the full solutions of Eqs. (21) as follows:

$$x(\theta) = e^{-\tilde{\zeta}(\theta/z)}(m_1 \cos\tilde{\eta}(\theta/z) + m_2 \sin\tilde{\eta}(\theta/z)) + A(\sin(\theta+\tilde{\tau}) - \sin(\theta+\tilde{\tau}_0)) + x_p(\theta) \tag{23a}$$

$$\dot{x}(\theta) = [e^{-\tilde{\zeta}(\theta/z)}/z][(m_2\tilde{\eta} - m_1\tilde{\zeta})\cos\tilde{\eta}(\theta/z) - (m_2\tilde{\zeta} + m_1\tilde{\eta})\sin\tilde{\eta}(\theta/z)] + A(\cos(\theta+\tilde{\tau}) - \cos(\theta+\tilde{\tau}_0)) + \dot{x}_p(\theta) \tag{23b}$$

$$y(\theta) = m_3 + m_4\theta - e_1\theta^2/2 \tag{23c}$$

$$\dot{y}(\theta) = m_4 - e_1\theta \tag{23d}$$

where $\tilde{\zeta} = \zeta + u$, $\tilde{\eta} = \sqrt{1+\nu-\tilde{\zeta}^2}$, $\tilde{\tau} = \delta - \tilde{\varphi}$, $\tilde{\tau}_0 = \delta_0 - \tilde{\varphi}$, $\tilde{\varphi} = \tan^{-1}[2\tilde{\zeta}z/(1+\nu-z^2)]$, $A = \sqrt{\frac{(1-z^2)^2+(2\zeta z)^2}{(1+\nu-z^2)^2+(2\zeta z+2uz)^2}}$, and the constants of integration m_j, j=1,2,3,4, are determined by the initial condition and parameters of the system.

Let $\tilde{\mathbf{X}}^* = (x^*,\dot{x}^*,\dot{y}^*,\tau^*)^{\mathrm{T}}$ denotes a fixed point of the Poincaré map (18), we choose the Poincaré section defined by

$$\sigma = \{(x,\dot{x},y,\dot{y},\theta) \in \mathrm{R}^4 \times \mathrm{S}, x = y, \dot{x} = \dot{x}_+, \dot{y} = \dot{y}_+\} \tag{24}$$

where R stands for the real space. S=R(mod 2π) means the periodicity of period T of the original system (3) and (4).

The following implicit equation is obtained from x=y in (24),

$$G(\theta,x,\dot{x},\dot{y},\tau) = x(\theta,x,\dot{x},\dot{y},\tau) - y(\theta,x,\dot{x},\dot{y},\tau) = 0 \tag{25}$$

with $G(\theta^*,x^*,\dot{x}^*,\dot{y}^*,\tau^*) = 0$, $\frac{\partial G}{\partial\theta}\big|_{(\theta^*,x^*,\dot{x}^*,\dot{y}^*,\tau^*)} \neq 0$.By virtue of the implicit function theorem, θ can be solved from Eq. (25)

$$\theta = \theta(x,\dot{x},\dot{y},\tau) \tag{26}$$

Then, we may obtain the Poincaré map,

$$\mathbf{X_{k+1}} = \mathbf{F}(\alpha,\boldsymbol{\rho},\mathbf{X_k}) = \begin{pmatrix} \hat{F}_1(\alpha,\boldsymbol{\rho},\theta(x_k,\dot{x}_k,\dot{y}_k,\tau_k),x_k,\dot{x}_k,\dot{y}_k,\tau_k) \\ \hat{F}_2(\alpha,\boldsymbol{\rho},\theta(x_k,\dot{x}_k,\dot{y}_k,\tau_k),x_k,\dot{x}_k,\dot{y}_k,\tau_k) \\ \hat{F}_3(\alpha,\boldsymbol{\rho},\theta(x_k,\dot{x}_k,\dot{y}_k,\tau_k),x_k,\dot{x}_k,\dot{y}_k,\tau_k) \\ \hat{F}_4(\alpha,\boldsymbol{\rho},\theta(x_k,\dot{x}_k,\dot{y}_k,\tau_k),x_k,\dot{x}_k,\dot{y}_k,\tau_k) \end{pmatrix} = \begin{pmatrix} F_1(\alpha,\boldsymbol{\rho},x_k,\dot{x}_k,\dot{y}_k,\tau_k) \\ F_2(\alpha,\boldsymbol{\rho},x_k,\dot{x}_k,\dot{y}_k,\tau_k) \\ F_3(\alpha,\boldsymbol{\rho},x_k,\dot{x}_k,\dot{y}_k,\tau_k) \\ F_4(\alpha,\boldsymbol{\rho},x_k,\dot{x}_k,\dot{y}_k,\tau_k) \end{pmatrix} \tag{27}$$

where $\mathbf{X_k} = (x_k,\dot{x}_k,\dot{y}_k,\tau_k)^{\mathrm{T}}$ consists of the state variables, the system parameter $\alpha \in \mathrm{R}$ denotes the bifurcation parameter, and the parameter vector $\boldsymbol{\rho} = (u,\nu)$ is the control gains.

3.2. Explicit bifurcation criterion of Hopf bifurcation in Poincaré map

In order to create the quasi-periodic impact motion of the original system via control, we need to design Hopf bifurcation of the Poincaré map in the close-loop system at a specified system parameter location to obtain a Hopf limit circle with desired dynamic properties. Our main work is to determine the gain vector $\boldsymbol{\rho}$. As mentioned above, it is difficult to search for the proper values of gains by scanning the parameter plane (u,ν) point by point to compute eigenvalues and check their properties. The explicit criterion of Hopf bifurcation for four-dimensional map without directly using eigenvalues of the Jacobian matrix [15] is employed to overcome the limitations of the classical critical criterion. The explicit criterion is formulated using a set of simple equalities or inequalities that consist of the coefficients of the characteristic equation derived from the Jacobian matrix. The control parameter mechanism of Hopf bifurcation for map may be explicitly formulated. As shown in the numerical example of the next section, one of the inequalities in the criterion might pick off half of parameter domain in the plane (u,ν) and the gains may be directly solved in terms of the equalities.

We give the explicit criterion of Hopf bifurcation [15] for four-dimensional map below.

Let the Jacobian matrix of map (27) at $\tilde{\mathbf{X}}^* = (x^*,\dot{x}^*,\dot{y}^*,\tau^*)^{\mathrm{T}}$ is $\mathbf{DF}(\alpha,\boldsymbol{\rho}) = \mathbf{A}(\alpha,\boldsymbol{\rho})$. The characteristic equation for matrix $\mathbf{A}(\alpha,\boldsymbol{\rho})$ is written as

$$P_\alpha(\lambda) = \lambda^4 + a_1\lambda^3 + a_2\lambda^2 + a_3\lambda + a_4 \tag{28}$$

where $a_i = a_i(\alpha,\boldsymbol{\rho})$, $i = 1,\ldots 4$.

Consider the following determinants

$$\Delta_1^{\pm}(\alpha,\boldsymbol{\rho}) = 1 \pm a_4$$

$$\Delta_2^{\pm}(\alpha,\boldsymbol{\rho}) = \left| \begin{pmatrix} 1 & a_1 \\ 0 & 1 \end{pmatrix} \pm \begin{pmatrix} a_3 & a_4 \\ a_4 & 0 \end{pmatrix} \right|$$

$$\Delta_3^{\pm}(\alpha,\boldsymbol{\rho}) = \left| \begin{pmatrix} 1 & a_1 & a_2 \\ 0 & 1 & a_1 \\ 0 & 0 & 1 \end{pmatrix} \pm \begin{pmatrix} a_2 & a_3 & a_4 \\ a_3 & a_4 & 0 \\ a_4 & 0 & 0 \end{pmatrix} \right|$$

Lemma 1. [15] *A Hopf bifurcation for map* (27) *occurs at* $\alpha=\alpha_c$ *if and only if the following conditions* (i)–(iv) *are satisfied.*

(i) $\Delta_3^-(\alpha_c,\boldsymbol{\rho})=0$
(ii) $P_{\alpha_c}(1)>0, \quad P_{\alpha_c}(-1)>0, \quad \Delta_3^+(\alpha_c,\boldsymbol{\rho})>0, \quad \Delta_1^\pm(\alpha_c,\boldsymbol{\rho})>0$
(iii) $\dfrac{d\Delta_3^-(\alpha_c,\boldsymbol{\rho})}{d\alpha}\neq 0$
(iv) $\cos(2\pi/m)\neq\psi, \quad \psi=1-0.5P_{\alpha_c}(1)\Delta_1^-(\alpha_c,\boldsymbol{\rho})/\Delta_2^+(\alpha_c,\boldsymbol{\rho}), \quad m=3,4,5\ldots$

Lemma 1 will be employed to determine the gains to trigger a quasi-periodic motion of the close-loop system at a specified system parameter location.

3.3. Existence of quasi-periodic impact motion of inertial impact shaker system

As an example, we choose the set of parameters ζ=0.01, β=0.8, μ=0.6, z=2 and take $\hat{R}$ as the bifurcation parameter (i.e., $\alpha=\hat{R}$). The original system at $\hat{R}=\hat{R}_c=0.8$ exhibits the stable periodic motion which is shown in the Poincaré section (24) as a stable fixed point (see Fig. 3).

Assumed that $\hat{R}=\hat{R}_c=0.8$ is the specified location at which the original system is required to transfer from the existing periodic motion to a stable quasi-periodic motion to be created. We thus design Hopf bifurcation of the Poincaré map (27) at $\hat{R}=\hat{R}_c$ by adjusting control parameters u and v appropriately.

According to the explicit criterion of Hopf bifurcation for map in Lemma 1, Maple software is employed to solve the equalities and inequalities (i)–(iv) to obtain the control parameter bifurcation plot (see Fig. 4).

In Fig. 4, the blank region denotes the parameter domain in which all inequalities in the conditions (ii) and (iii) of Lemma 1 are satisfied whereas in the gray region III and IV at least one inequality fails. The open domain I that is surrounded with the black arc AD, the blue lines AB and L_1, and the green line BC, stands for the stability region in which

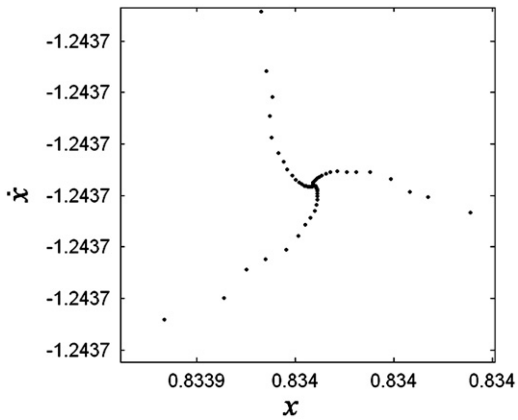

Fig. 3. Stable periodic motion projected to the Poincaré section.

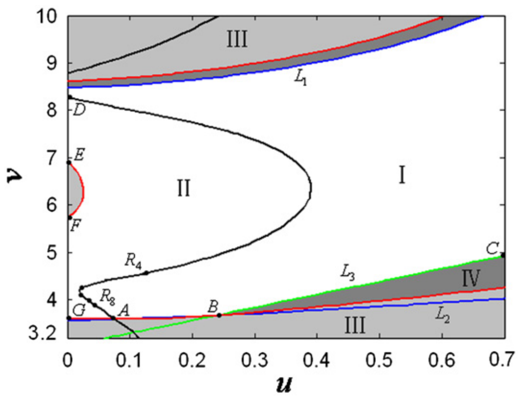

Fig. 4. Control parameter bifurcation diagram.

$\Delta_3^-(\alpha_c,\boldsymbol{\rho})>0$ and all of the inequalities in the condition (ii) hold. The other blank region II, surrounded by the black arc *AD* and the red arcs *EF* and *AG*, represents the potential parameter region where there may exist the quasi-periodic solutions near the *AD*. The black arc *AD* consists of the parameter points that satisfy $\Delta_3^-(\alpha_c,\boldsymbol{\rho})=0$ as well as the inequality constraints in (ii) and (iii). The black points from R_4 to R_8 on the arc *AD* represent the resonance points with $m=4,5\ldots8$ in (iv), respectively. To design Hopf bifurcation in the nonresonance case, the gains should be chosen on the arc *AD* but away from these resonance points.

We choose one of the points on the open arc *AD* with $(u,v)=(0.03,4.2955731047)$ as the theoretical values of gains. It follows from Lemma 1 that a Hopf bifurcation of map (27) occurs at $\hat{R}_c=0.8$. In other words, a quasi-periodic impact motion of the inertial impact shaker system, as a created nonlinear solution, is triggered at $\hat{R}_c=0.8$ via control.

In summary, it is convenient and efficient to create Hopf bifurcations on the basis of the control parameter bifurcation plot where the stability domain and the bifurcation domain in the parameter plane are clear.

3.4. Stability of quasi-periodic impact motion of inertial impact shaker system

In this subsection, we shall discuss the stability of the bifurcating solutions (Hopf limit circle) of the map (27) (or the quasi-periodic impact motion of the inertial impact shaker system under control). The stability of Hopf limit circle depends on the nonlinear property of the map (27). Some methodologies such as the center manifold reduction and normal form theory [16,17] and frequency domain approach [13], are capable for determining the stability analytically. Here the method we use is based on the theories of discrete system by Kuznetsov [16].

First, we transform the fixed point $\tilde{\mathbf{X}}^*$ and bifurcation point $\hat{R}_c$ to the origin point by the change of variables,

$$\mathbf{Y_k}=\mathbf{X_k}-\tilde{\mathbf{X}}^*,\quad \varepsilon=\hat{R}-\hat{R}_c \tag{29}$$

Under the change of variables (29), the map (27) becomes

$$\mathbf{Y_{k+1}}=\tilde{\mathbf{F}}(\varepsilon,\mathbf{Y_k}) \tag{30}$$

We expand the map (30) as Taylor series in the variable $\mathbf{Y_k}$

$$\mathbf{Y_{k+1}}=\mathbf{A}\mathbf{Y_k}+\frac{1}{2}\mathbf{B}(\mathbf{Y_k},\mathbf{Y_k})+\frac{1}{6}\mathbf{C}(\mathbf{Y_k},\mathbf{Y_k},\mathbf{Y_k})+o(\|\mathbf{Y_k}\|^4) \tag{31}$$

where $\mathbf{A}$ is the Jacobian matrix at the fixed point $\tilde{\mathbf{X}}^*$ at $\hat{R}=\hat{R}_c$. $\mathbf{A}$ has a complex conjugate pairs of eigenvalues $\lambda(0)$ and $\overline{\lambda}(0)$ on the unit circle, the other eigenvalues are strictly inside the unit circle.

The stability of Hopf limit circle in map (31) can be analyzed by the sign of $\phi(0)$ in equation below,

$$\phi(0)=\mathrm{Re}\left(\frac{\overline{\lambda}(0)g_{21}}{2}\right)-\mathrm{Re}\left(\frac{(1-2\lambda(0))\overline{\lambda}^2(0)}{2(1-\lambda(0))}g_{20}g_{11}\right)-\frac{1}{2}|g_{11}|^2-\frac{1}{4}|g_{02}|^2 \tag{32}$$

where $g_{20}=\langle\mathbf{p},\mathbf{B}(\mathbf{q},\mathbf{q})\rangle$, $g_{11}=\langle\mathbf{p},\mathbf{B}(\mathbf{q},\overline{\mathbf{q}})\rangle$, $g_{02}=\langle\mathbf{p},\mathbf{B}(\overline{\mathbf{q}},\overline{\mathbf{q}})\rangle$, $g_{21}=\langle\mathbf{p},\mathbf{C}(\mathbf{q},\mathbf{q},\overline{\mathbf{q}})\rangle+2\langle\mathbf{p},\mathbf{B}(\mathbf{q},(\mathbf{I}-\mathbf{A})^{-1}\mathbf{B}(\mathbf{q},\overline{\mathbf{q}}))\rangle+\langle\mathbf{p},\mathbf{B}(\overline{\mathbf{q}},(\lambda^2(0)\mathbf{I}-\mathbf{A})^{-1}\mathbf{B}(\mathbf{q},\mathbf{q}))\rangle+\frac{\overline{\lambda}(0)(1-2\lambda(0))}{1-\lambda(0)}\langle\mathbf{p},\mathbf{B}(\mathbf{q},\mathbf{q})\rangle\langle\mathbf{p},\mathbf{B}(\mathbf{q},\overline{\mathbf{q}})\rangle-\frac{2}{1-\overline{\lambda}(0)}|\langle\mathbf{p},\mathbf{B}(\mathbf{q},\overline{\mathbf{q}})\rangle|^2-\frac{\lambda(0)}{\lambda^3(0)-1}|\langle\mathbf{p},\mathbf{B}(\overline{\mathbf{q}},\overline{\mathbf{q}})\rangle|^2$It follows from (32) that $\phi(0)=-1.83<0$. According to the criterion of stability [16], the system has a stable invariant cycle (quasi-periodic motion) near $\hat{R}_c$ under the chosen control gains.

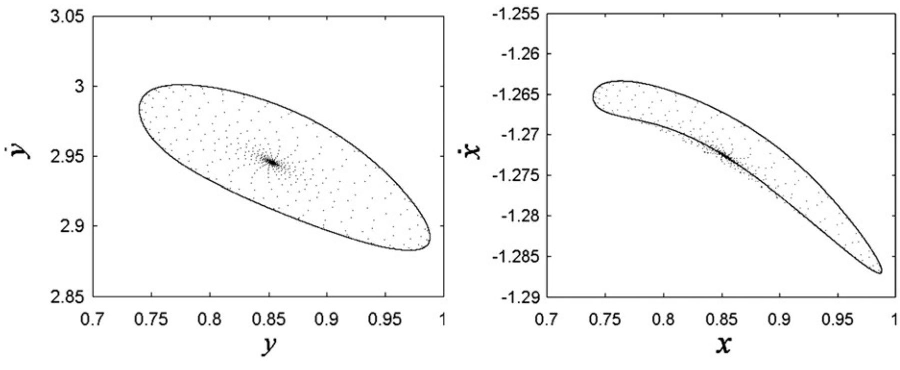

Fig. 5. Stable invariant cycle in the Poincaré section.

3.5. Simulations

At the set of parameters of the original system $\zeta = 0.01$, $\beta = 0.8$, $\mu = 0.6$, $z = 2$, $u = 0.03$, $\nu = 4.3$, Fig. 5 shows the stable Hopf limit circle by setting $\hat{R} = \hat{R}_c + 0.01 = 0.81$, which represents the created quasi-periodic impact motion of the inertial impact shaker system.

4. Conclusions

In this paper, the linear feedback control method is proposed for anti-control of quasi-periodic impact motion in inertial impact shaker system based on the bifurcation theory of maps. An explicit criterion of Hopf bifurcation without directly using eigenvalues is utilized to overcome the limitation of the classical critical criterion of Hopf bifurcation in control design. The stable quasi-periodic impact motion of the original system is created in a pre-specified parameter location by adjusting control parameter appropriately.

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